Forecasting Quarterly Exchange Rates Using Fuzzy Time Series: A Catalyst for Business Projections Amidst Global Pandemic

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Abstract

In the phase of global pandemic, a knowledge of exchange rates movement remains an indispensable tool for international business projections. However, projections amidst global pandemic requires forecasting methods that works adequately with an incomplete information for which Fuzzy Time Series method is appropriate. Hence, this paper proposes a novel fuzzy time series model for forecasting quarterly average Inter-bank Foreign Exchange Market (IFEM) rates of Nigeria Naira against United States Dollar derived from monthly average rates over the periods 2004 to 2020 made available at www.cbn.gov.ng/rates/exrates.asp by the Central Bank of Nigeria. *The effectiveness of the proposed model based on performance metric (MAPE) indicates that the proposed fuzzy time series method is suitable for forecasting the Naira's parity with the United States Dollar.*

Keywords: *Foreign Exchange Rate; Fuzzy; Naira; Time Series; USD*

1. Introduction

The current global financial crisis due to COVID-19 pandemic has created more fears in the international business space particularly in the emerging economy. In Nigeria, this fear is further aggravated by dwindling foreign exchange (Forex) rates particularly in the parallel market. The turmoil in the Forex market may be attributed to crash in global crude oil prices among other factors in the early phase of the COVID-19 pandemic and the slow price appreciation currently the global oil market is experiencing. This scenario is posing great challenges in the international business decisions. The key interests in such atmosphere most not be in short of a framework for forecasting the movement of the future Forex indices. One indispensable tool in this regard is the conventional Time Series models. Autoregressive (AR), Autoregressive Moving Average (ARMA) and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) are the common conventional Time Series models found in the literature of forecasting foreign exchange rates. However, the application of these methods are predetermined by some assumptions, for instance (Leu, Lee, & Jou, 2009) opined that in applying ARMA method one needs to make sure that the residue is normally distributed. Additionally, forecasting problems in which the historical data are not in precise sense cannot be handled by these methods. Furthermore, in the words of (Korol, 2014) the fluctuation of exchange rates might not be understood completely due to a lack of information and the movement of exchange rates is affected by many factors (economic, political, psychological, etc.) that cannot be precisely and unambiguously defined. Therefore, the search for technique that is insensitive to these constraints and that might work efficiently in forecasting exchange rates for which Fuzzy Time Series (FTS) is ideal is inevitable.

The concept of Fuzzy Time Series was built on the characteristics of fuzzy set theory, the theory was introduced by Zadeh (Ghosh, Chowdhury, & Prajneshu, 2016; Alves et al., 2018) and it has been applied to several diverse areas (Sasu, 2010). Accordingly, Song and Chissom proposed the concept of FTS as noted in (Hosseini, Fard, & Baboli, 2011; Olatayo & Taiwo, 2014). Similarly, (Garg, Sufyan Beg, & Ansari, 2013) noted that substantial work has been done on forecasting problems using fuzzy time series since it preposition. Areas such as university enrolments, stock index forecasting, market assets, economic indicators, exchange rate, electric load, temperature forecasting and tourism forecast are notable areas among many others that were subjected to fuzzy time series application over the years.

Fuzzy Time Series forecasting methodology consists of defining universe of discourse (UOD), fuzzification of time series data points, assigning relationships between consecutive data points and defuzzification to get back the forecasting results in real domain (Pal & Kar, 2019).

In this paper, we aim to model the amount of Nigeria Naira (NGN) that one requires to buy one United States Dollar (USD) using fuzzy time series approach based on quarterly parity rates between the two currency for the period 2004 to 2020 second quarter. To make the NGN to USD currency rate forecasting methodology and the data analyses self-acquainted, definitions of related concepts were reproduced under review of literature.

2. Review of Literature

Parity forecasting between currencies has been researched in many studies over years, this claim is visible in the work of (Tlegenova, 2015), who opined that there are lots of works done on time series based on prediction modelling of foreign currency rates in literature. Fuzzy time series methodology is one popular time series approaches in the literature that has attracted the attention of researchers in forecasting foreign exchange rates. For instance, (Efendi, Ismail, & Deris, 2013) noted that the fuzzy logic methods are found to be suitable due to its ability in pattern recognition and handling of the non-probabilistic uncertainties type in the forex data. Similarly, (Korol, 2014) created a model with high efficiency based on the fuzzy logic model that uses average quarterly exchange rates of JPY/USD, GBP/USD and CHF/USD as experimental data. Additionally, (Boiroju &

Rao, 2015), developed fuzzy time series model for daily exchange rate of the Indian rupees against US Dollar. Furthermore, (Leu et al., 2009) and (Permana & Fitri, 2020) respectively studied the currency rates between Riyal to Rupiah and New Taiwan Dollar (NTD) to USD using fuzzy time series model.

A reproduced summary of definitions for fuzzy time series concept found in (Song & Chissom, 1993), (Chou, 2016), (Garg et al., 2013), (Lee & Chou, 2004), (Sah & Degtiarev, 2005) are accordingly provided below:

Definition 1

Let $Y(t)$ ($t = \ldots, 0, 1, 2, \ldots$), a subset of $\mathbb{R}^1$, be the universe of discourse on which fuzzy sets $f_i(t)(i = 1, 2, \ldots)$ are defined and $F(t)$ is a collection of $f_i(t)(i = 1, 2, \ldots)$. Then $F(t)$ is called fuzzy time series on $Y(t)(t = \ldots, 0, 1, 2, \ldots)$.

Definition 2

The universe of discourse $U = [D_L, D_u]$ is defined such that:

$D_L = D_{min} - st_{\alpha,n}/\sqrt{n}$ and $D_u = D_{max} + st_{\alpha,n}/\sqrt{n}$, when $n \leq 30$ or $D_L = D_{min} - \sigma z_\alpha/\sqrt{n}$ and $D_u = D_{max} + \sigma z_\alpha/\sqrt{n}$, when $n > 30$;

Where: $t_{\alpha,n}$ is $100(1-\alpha)$ percentile of the t distribution,

z_α is $100(1-\alpha)$ percentile of the standard normal distribution

s & σ denotes sample and population standard deviations respectively.

D_{min} and D_{max} are respectively minimum and maximum values of the data in question

Definition 3

Let $\mathrm{F}(\mathrm{t}-1) = A_i$ and $F(t) = A_j$. Relationship between two consecutive observations, $F(t)$ and $F(t - 1)$, referred to as a fuzzy logical relationship (FLR), can be denoted by $A_i \rightarrow A_j$ where A_i is called the left-hand side (LHS) and A_j is the right-hand side (RHS) of the FLR.

Definition 4

Assuming that there are m linguistic values under consideration, let $A_i$be the fuzzy number that represents the ith linguistic value of the linguistic variable, where $1 \leq i \leq m$. The support of A_i is defined to be:

$$D_l + (i-1)\frac{D_u - D_l}{m}, \quad D_l + \frac{i(D_u - D_l)}{m}, 1 \leq i \leq m-1$$
$$D_l + (i-1)\frac{D_u - D_l}{m}, \quad D_l + \frac{i(D_u - D_l)}{m}, i = m \qquad (1)$$

Definition 5

If there exists a fuzzy relationship $\mathrm{R}(\mathrm{t}, \mathrm{t}-1)$, such that $\mathrm{F}(\mathrm{t}) = \mathrm{F}(\mathrm{t}-1) x \mathrm{R}(\mathrm{t}, \mathrm{t}-1)$, where symbol x is an operator, then $\mathrm{F}(\mathrm{t})$ is said to be caused by $\mathrm{F}(\mathrm{t}-1)$. The existing relationship between $\mathrm{F}(\mathrm{t})$ and $\mathrm{F}(\mathrm{t}-1)$ can be denoted by the expression $\mathrm{F}(\mathrm{t}-1) \rightarrow \mathrm{F}(\mathrm{t})$

3. Methodology

The stepwise outline of the proposed forecasting process in accordance with the (Pal & Kar, 2019) description of fuzzy time series forecasting methodology earlier noted for quarterly average Inter-bank Foreign Exchange Market (IFEM) rates of Nigeria Naira against United States Dollar is presented as follows:

Step 1. Determine universe of discourse on IFEM rates of NGN to USD as, $U = [\ D_L, D_U]$

Step 2. Partition the universe of discourse U into several even and equal length intervals. Thus, $U = \bigcup_{i=1}^{m} u_i$, letting m the number of disjoint intervals.

Step 3. Determine the fuzzy sets A_i: some linguistic values represented by fuzzy sets of the interval of the universe of discourse such that its membership function is as follows:

$$u_{A_i}(x) = \begin{cases} 1\ for\ x \in \left[D_l + (i-1)\frac{D_u - D_l}{m},\ D_l + \frac{i(D_u - D_l)}{m}\right), \\ \quad where 1 \leq i \leq m-1; \\ 1\ for\ x \in \left[D_l + (i-1)\frac{D_u - D_l}{m},\ D_l + \frac{i(D_u - D_l)}{m}\right), \\ \quad i = m; \\ o\ otherwise. \end{cases} \qquad (2)$$

Step 4. Then, $\mathrm{F}(\mathrm{t}) = A_i$, if $IFEM(t) \ni \mathrm{supp}(A_i)$, where $\mathrm{supp}(.)$ denotes support. Note that $\forall\, t \,\exists\, k/k = Q1, Q2, Q3\ \&\ Q4$

Step 5. Identify fuzzy linguistic relationships (FLR's) among linguistic time series values, $A_i \rightarrow A_j$.

Step 6. Establish fuzzy relationship groups (FLRGs): identification of $A_i \to A_j$ (FLR's) having the same LHS.

Step 7. Establish forecasting rule.

Step 8. Determine the forecast values based on the established rule.

Step 9. Forecast evaluation using performance metrics

4. Experimental Results

To test the proposed model, the empirical analysis of the Inter-bank Foreign Exchange Market (IFEM) rates of Nigeria Naira against United States Dollar in accordance with the outlined procedures presented in section 3 is demonstrated in this section. The selected data set for this demonstration includes 66 records of quarterly IFEM rates of NGN against USD derived from monthly average rates over the periods January, 2004 to June, 2020. A comparative assessment of the forecasted values alongside the observed records is given at the end of this section. Additionally, the evaluation of the proposed model in terms of performance metric (MAPE) is also reported.

Onward in the section, let $Q_{t_{FX}}$ represent the quarterly IFEM rates of NGN against USD data set, and in order to improve accuracy of our forecast, natural logarithm of the $Q_{t_{FX}}$ is taken. Hence;

$$ln(Q_{t_{FX}}) = q_{t_{fx}} \equiv \exp(q_{t_{fx}}) = Q_{t_{FX}}$$

From our data set, the following statistics from table 1 were observed: $n = 66, s = 0.365$, $q_{t_{fxmin}} = 4.308$, and $q_{t_{fxmax}} = 5.889$. The forecast values derivation in line with the section 3 outlines can be demonstrated as follows:

Step 1. From definition 2, the universe of discourse $U = [D_L, D_u]$, since $n > 30$, a $100(1-\alpha)$ percentile of the standard normal distribution is considered. Therefore ,letting $\alpha = 0.05$, $z_{0.05} = 1.96$, $D_L = q_{t_{fxmin}} - \sigma z_\alpha/\sqrt{n} \cong 4.220$ and $D_u = q_{t_{fxmax}} + \sigma z_\alpha/\sqrt{n} \cong 5.977$. Thus, $U = [4.220, 5.977]$.

Step 2. The partitioning of the universe of discourse into m even and equal length intervals is as follows: m is set to 7, because it is the usual practice as noted in (Ghosh, Chowdhury, & Prajneshu, 2016). Therefore, $u_1 = [4.220, 4.471)$, $u_2 = (4.471, 4.722)$, $u_3 = (4.722, 4.973)$, $u_4 = (4.973, 5.224)$, $u_5 = (5.224, 5.475)$, $u_6 = (5.475,5.726)$, and $u_7 = (5.726, 5.977]$.

Step 3. Assuming that the followings linguistic values describe the quarterly Naira-USD parity rates: very very low, very low, low, no change, high, very high, very very high such that the membership function is given as:

$$u_{A_i}(x) = \begin{cases} 1\, for\ q_{t_{fx}} \in [4.220 + (i-1)(0.251),\ 4.220 + i(0.251), \\ where 1 \le i \le m-1; \\ 1\, for\ q_{t_{fx}} \in [4.220 + (i-1)(0.251),\ 4.220 + i(0.251)] \\ i = m\,; \\ 0\ otherwise. \end{cases}, \qquad (3)$$

Where A_1 = very very low, A_2 = very low, A_3 = low, A_4 = no change, A_5 = high, A_6 =very high and A_7 = very very high. Hence, the supports are: supp$(A_1) = [4.220, 4.471)$, supp$(A_2) = (4.471, 4.722)$, supp$(A_3) = (4.722, 4.973)$, supp$(A_4) = (4.973, 5.224)$, supp$(A_5) = (5.224, 5.475)$, supp$(A_6) = (5.475,5.726)$, and supp$(A_7) = (5.726, 5.977]$.

The mid-points of the intervals computed as average of the lower and upper limits of each linguistic value are obtained as: $\bar{u}_1 = 4.346$, $\bar{u}_2 = 4.597$, $\bar{u}_3 = 4.878$, $\bar{u}_4 = 5.099$, $\bar{u}_5 = 5.350$, $\bar{u}_6 = 5.601$ and $\bar{u}_7 = 5.852$.

Step 4. The Fuzzy time series F(t) on $Q_{t_{FX}}$ is A_i, if $q_{t_{fx}} \in$ supp(A_i). Thus, $F(Q_{12004}) = A_3$, $F(Q_{22004}) = A_3, \ldots, F(Q_{22020}) = A_7$. These transformation of the $Q_{t_{FX}}$ into fuzzy values is given in Table 1.

Step 5. In accordance with definition 3, the Fuzzy Logical Relationships are as follows: $A_3 \to A_3$, $A_3 \to A_3, \ldots$ and $A_7 \to A_7$ as shown in table 1.

Table 1: Fuzzified Values for IFEM Rates Data

Year/ Quarter	Actual IFEM Rates	Fuzzified IFEM Rates	IFEM Rates FLRs	Year/ Quarter	Actual IFEM Rates	Fuzzified IFEM Rates	IFEM Rates FLRs
2004Q1	4.915	A_3		2012Q2	5.069	A_4	$A_4 \to A_4$
2004Q2	4.907	A_3	$A_3 \to A_3$	2012Q3	5.071	A_4	$A_4 \to A_4$
2004Q3	4.896	A_3	$A_3 \to A_3$	2012Q4	5.059	A_4	$A_4 \to A_4$
2004Q4	4.893	A_3	$A_3 \to A_3$	2013Q1	5.061	A_4	$A_4 \to A_4$
2005Q1	4.891	A_3	$A_3 \to A_3$	2013Q2	5.067	A_4	$A_4 \to A_4$
2005Q2	4.895	A_3	$A_3 \to A_3$	2013Q3	5.084	A_4	$A_4 \to A_4$
2005Q3	4.902	A_3	$A_3 \to A_3$	2013Q4	5.070	A_4	$A_4 \to A_4$
2005Q4	4.873	A_3	$A_3 \to A_3$	2014Q1	5.093	A_4	$A_4 \to A_4$
2006Q1	4.862	A_3	$A_3 \to A_3$	2014Q2	5.089	A_4	$A_4 \to A_4$
2006Q2	4.856	A_3	$A_3 \to A_3$	2014Q3	5.090	A_4	$A_4 \to A_4$
2006Q3	4.855	A_3	$A_3 \to A_3$	2014Q4	5.148	A_4	$A_4 \to A_4$
2006Q4	4.855	A_3	$A_3 \to A_3$	2015Q1	5.253	A_5	$A_4 \to A_5$
2007Q1	4.855	A_3	$A_3 \to A_3$	2015Q2	5.283	A_5	$A_5 \to A_5$
2007Q2	4.850	A_3	$A_3 \to A_3$	2015Q3	5.283	A_5	$A_5 \to A_5$
2007Q3	4.840	A_3	$A_3 \to A_3$	2015Q4	5.283	A_5	$A_5 \to A_5$
2007Q4	4.791	A_3	$A_3 \to A_3$	2016Q1	5.283	A_5	$A_5 \to A_5$
2008Q1	4.765	A_3	$A_3 \to A_3$	2016Q2	5.340	A_5	$A_5 \to A_5$
2008Q2	4.768	A_3	$A_3 \to A_3$	2016Q3	5.714	A_6	$A_5 \to A_6$
2008Q3	4.768	A_3	$A_3 \to A_3$	2016Q4	5.721	A_7	$A_6 \to A_7$
2008Q4	4.815	A_3	$A_3 \to A_3$	2017Q1	5.722	A_7	$A_7 \to A_7$
2009Q1	4.591	A_2	$A_3 \to A_2$	2017Q2	5.723	A_7	$A_7 \to A_7$
2009Q2	4.308	A_1	$A_2 \to A_1$	2017Q3	5.723	A_7	$A_7 \to A_7$
2009Q3	5.029	A_4	$A_1 \to A_4$	2017Q4	5.723	A_7	$A_7 \to A_7$
2009Q4	5.013	A_4	$A_4 \to A_4$	2018Q1	5.723	A_7	$A_7 \to A_7$
2010Q1	5.014	A_4	$A_4 \to A_4$	2018Q2	5.723	A_7	$A_7 \to A_7$
2010Q2	5.018	A_4	$A_4 \to A_4$	2018Q3	5.724	A_7	$A_7 \to A_7$
2010Q3	5.019	A_4	$A_4 \to A_4$	2018Q4	5.726	A_7	$A_7 \to A_7$
2010Q4	5.022	A_4	$A_4 \to A_4$	2019Q1	5.726	A_7	$A_7 \to A_7$
2011Q1	5.034	A_4	$A_4 \to A_4$	2019Q2	5.727	A_7	$A_7 \to A_7$
2011Q2	5.046	A_4	$A_4 \to A_4$	2019Q3	5.727	A_7	$A_7 \to A_7$
2011Q3	5.036	A_4	$A_4 \to A_4$	2019Q4	5.727	A_7	$A_7 \to A_7$
2011Q4	5.077	A_4	$A_4 \to A_4$	2020Q1	5.748	A_7	$A_7 \to A_7$
2012Q1	5.070	A_4	$A_4 \to A_4$	2020Q2	5.889	A_7	$A_7 \to A_7$

Step 6. From the FLRs identified in step 5, ten FLR groups were derived by identifying FLRs having the same LHSs. These ten groups are as follows: $G_1: A_1 \to A_4$, $G_2: A_2 \to A_1$, $G_3: A_3 \to A_2$, $G_4: A_3 \to A_3$, $G_5: A_4 \to A_4$, $G_6: A_4 \to A_5$, $G_7: A_5 \to A_5$, $G_8: A_5 \to A_6$, $G_9: A_6 \to A_7$ and $G_{10}: A_7 \to A_7$.

Step 7. The proposed forecasting rule for the $Q_{t_{FX}}$ data set is as follows:

a) Let $F(t) = A_i$

a) Select FLR groups where fuzzy value A_i is a transition.

b) Define: 1) $Z_L = \{y/ y \text{ is LHS of the selected FLRG}(s)\}$

2) $Z_R = \{y/ y \text{ is RHS of the selected FLRG}(s)\}$

c) Let $W = Z_L \cup Z_R$

To demonstrate this rule, let $t = 2004Q1$, this implies that $F(t) = A_3$. Thus, the FLRGs where the fuzzy value A_3 is transition are $G_3: A_3 \rightarrow A_2$ and $G_4: A_3 \rightarrow A_3$. The set of LHS and RHS of the selected FLRGs are respectively:

$$Z_L = \{A_3, A_3\}$$
$$Z_R = \{A_2, A_3\}$$
$$W = \{A_2, A_3\}$$

Step 8. The in-sample forecast value is: $\hat{q}_{t_{fx}} = \frac{1}{l}(X\mu_{(t)}^p)$,

where: $\mu_{(t)}^p = (\bar{u}_{11}\ \bar{u}_{21} \ldots \bar{u}_{l1})$, an $(l\ x\ 1)$ matrix,

$\bar{u}_{i1}$ is mid-point of A_i, such that $A_i \in W, i = 1, 2, \ldots m$,

X, is an $(1\ x\ l)$ matrix with unit elements, $l \leq m$,

$\forall\, t \,\exists\, k/k = Q1, Q2, Q3\ \&\ Q4$

Therefore, for $t = 2004Q1$, $\mu_{(t)}^p = \begin{pmatrix} 4.597 \\ 4.878 \end{pmatrix}$, $X = (1 \quad 1)$;

Hence, $\hat{q}_{2004Q1_{fx}} = 4.738$; the forecasting results based on this rule for the periods in question are presented in table 2.

Table 2: Forecasted IFEM rates

Year/Quarter	Forecasted IFEM Rates	Year/Quarter	Forecasted IFEM Rates	Year/Quarter	Forecasted IFEM Rates
2004Q1	4.738	2009Q3	4.932	2015Q1	5.350
2004Q2	4.738	2009Q4	4.932	2015Q2	5.350
2004Q3	4.738	2010Q1	4.932	2015Q3	5.350
2004Q4	4.738	2010Q2	4.932	2015Q4	5.350
2005Q1	4.738	2010Q3	4.932	2016Q1	5.350
2005Q2	4.738	2010Q4	4.932	2016Q2	5.350
2005Q3	4.738	2011Q1	4.932	2016Q3	5.601
2005Q4	4.738	2011Q2	4.932	2016Q4	5.727
2006Q1	4.738	2011Q3	4.932	2017Q1	5.727
2006Q2	4.738	2011Q4	4.932	2017Q2	5.727
2006Q3	4.738	2012Q1	4.932	2017Q3	5.727
2006Q4	4.738	2012Q2	4.932	2017Q4	5.727
2007Q1	4.738	2012Q3	4.932	2018Q1	5.727
2007Q2	4.738	2012Q4	4.932	2018Q2	5.727
2007Q3	4.738	2013Q1	4.932	2018Q3	5.727
2007Q4	4.738	2013Q2	4.932	2018Q4	5.727
2008Q1	4.738	2013Q3	4.932	2019Q1	5.727
2008Q2	4.738	2013Q4	4.932	2019Q2	5.727
2008Q3	4.738	2014Q1	4.932	2019Q3	5.727
2008Q4	4.738	2014Q2	4.932	2019Q4	5.727
2009Q1	4.607	2014Q3	4.932	2020Q1	5.727
2009Q2	4.681	2014Q4	4.932	2020Q2	5.727

Step 9. Forecast evaluation

Statistical measures of the Actual IFEM (AIFEM) and Forecasted IFEM (FIFEM) values were presented in Table 2 in order to evaluate the performance of our proposed model. It can be realised that the actual and forecasted values statistics are approximately equal with slight differences. A pictorial comparison of these measures is shown in figure 1 and figure 2 to further highlight the accuracy of the proposed model. Furthermore, the MAPE value consolidated this statement of equality of the these measures as the value of the metric is within ten percent error, which is an indication of high accuracy as noted by (Akincilar, TemIz, & ŞahIn, 2011).

Table 2: Descriptive Statistics of Actual and Forecasted IFEM Rates

Measures	Max	Min	Mean	S	MAPE
IFEM	5.889	4.308	5.162	0.365	
Forecast	5.727	4.607	5.093	0.396	1.874

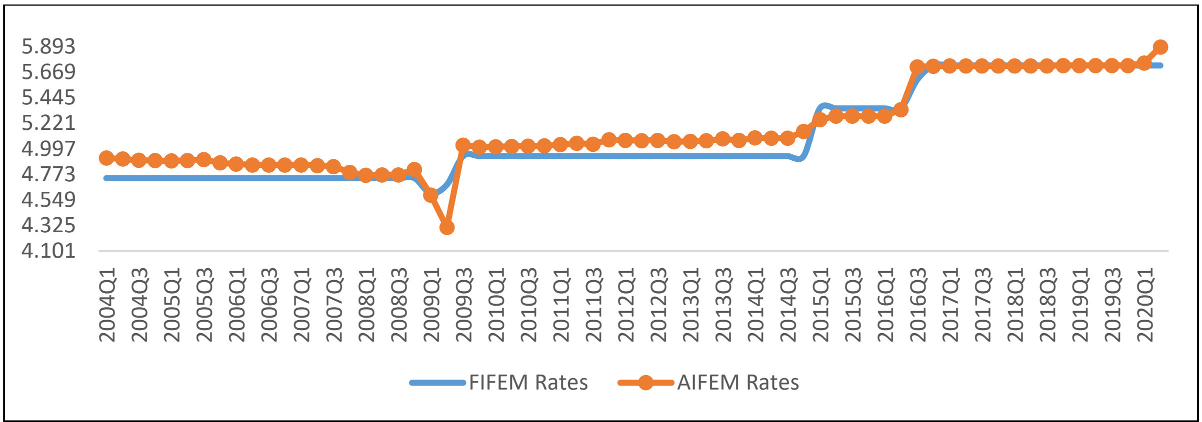

Fig. 1: AIFEM and FIFEM rates trends

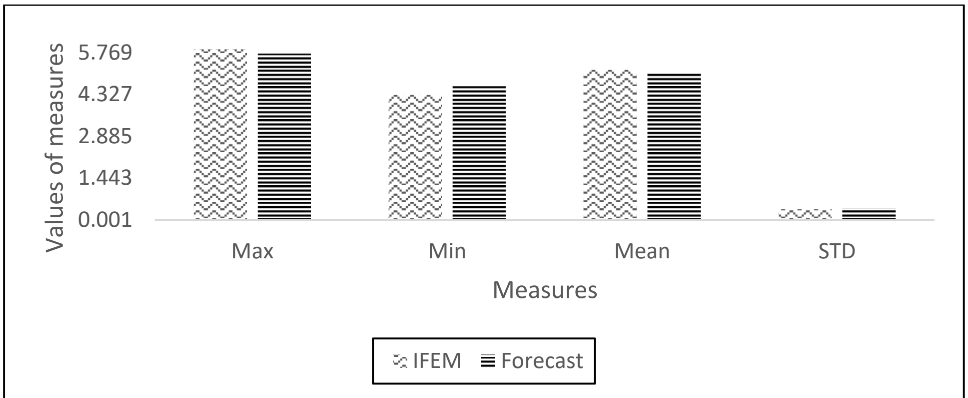

Fig. 2: Comparative view of descriptive statistics of Actual and Forecasted IFEM Rates

5. Conclusion

The development of optimal business decisions is a challenging task due to its sensitivity to the movement of foreign exchange rates particularly in an emerging economy. Therefore, the search for currency rates forecasting framework in the midst of global pandemic cannot be over emphasized. This paper proposes a fuzzy time series forecasting model for quarterly average Inter-Bank Foreign Exchange Market (IFEM) rates of Nigeria

Naira against United States Dollar. The results of our analysis yielded a simplified Naira against US Dollar parity rates forecasting model with a MAPE (Mean Absolute Percentage Error) value of 1.87%, indicating that the model worth recommendation for prediction practice. However, the application of the proposed model may be biased due to frequent exchange rate policies, hence, need for caution.

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