Mathematical Modeling of Magneto Pulsatile Blood Flow Through a Porous Medium with A Heat Source

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Abstract

In this current investigation, we introduce a mathematical framework to describe the behavior of non-Newtonian blood flow in a non-Darcy porous medium under the influence of a magnetic field, heat source, and Joule effect. The magnetic field is uniformly oriented perpendicular to the porous surface. To tackle the governing nonlinear partial differential equations, we employ the explicit finite difference method (FDM) for numerical solution. We delve into the impact of several crucial parameters, including the Reynolds number, hydro-magnetic parameter, Forchheimer parameter, Darcie parameter, Prandtl number, Eckert number, heat source parameter, and Schmidt number. Through the visualization of graphical representations, we analyze how these parameters affect the velocity, temperature, and concentration profiles. This research holds practical relevance in fields such as surgical procedures, industrial materials processing, and diverse heat transfer applications.

Keywords: heat source, porous medium and magnetohydrodynamic blood flow.

1.0 introduction

The circulation of blood within arteries represents a fundamental physiological phenomenon, drawing significant interest from the biomedical research community, physiologists, and clinicians. An intriguing facet of this phenomenon is the alteration in blood flow characteristics induced by the application of an external magnetic field, a topic that has become a focal point of intensive research in recent years.The roots of mathematical modeling in bio-fluid engineering, particularly in the context of heat transfer, can be traced back to the late 1940s when Pennes (1984) published a seminal paper laying the groundwork for understanding conduction heat transfer within tissue. The study of pulsatile fluid flow coupled with heat transfer has manifold applications, particularly within the realms of mechanical and industrial thermal engineering systems.Baish (1990) delved into the intricacies of heat transport within countercurrent blood vessels, even in the presence of arbitrary pressure gradients. Meanwhile, Consiglieri et al. (2003) and Davalos et al. (2003) engaged in comprehensive theoretical examinations of the heat convection coefficient within large blood vessels. Shrivastava et al. (2005) conducted an analytical investigation of heat transfer within finite tissue, considering the presence of two blood vessels and uniform Dirichlet boundary conditions.All of these investigations were limited to the realm of Newtonian blood flow models. While numerous studies have explored Newtonian models grounded in the Navier-Stokes equations, it is essential to acknowledge the rheological properties inherent in biological fluids like blood, plasma, and bile. Developing a non-Newtonian model is imperative to enhance the accuracy of results in the study of physiological fluids.Given the pulsatile nature of blood circulation driven by the heart's pumping action in the human system, it becomes crucial to account for this characteristic. Skalak and Chien (1982) conducted a study examining non-Newtonian blood flow, specifically considering erythrocytes as soft tissues. For a comprehensive overview of various rheological models pertaining to blood, Cokelet (1972) has provided an excellent summary.

In the mid-1980s, engineers began exploring the impact of magnetic fields on blood flow. Their primary objectives were to harness the principles of magneto-hydrodynamics (MHD) to regulate blood flow velocities during surgical procedures and to investigate the consequences of magnetic fields on blood circulation, particularly concerning astronauts and individuals residing near electromagnetic (EM) towers.The presence of iron oxides within the hemoglobin molecule, as demonstrated by Takeuchi et al. (1995), has been revealed to impart robust magnetic properties to blood. In oxygenated conditions, blood exhibits diamagnetic characteristics, while in deoxygenated states, it behaves as a paramagnetic fluid. Numerous studies have also addressed the topic of heat transfer within bio-magnetic fluid flows. Notable examples include the work of Tzirtzilakis and Tanoudis (2003), which examined bio-magnetic convective heat transfer over a stretching surface, and the research conducted by Louckopoulos and Tzirtzilakis (2004) on bio-magnetic flow and heat transfer in a parallel-plate system.

The inclusion of a porous medium in the context of fluid flow introduces a more physically realistic dimension to the study. This approach finds applicability in modeling phenomena within blood vessels and pulmonary systems, where factors like fatty deposits and artery blockages come into play. Typically, the Darcy model is the most commonly employed framework for representing porous conditions. However, under conditions characterized by higher pressure gradients and highly porous regimes where inertial effects outweigh viscous effects, the Darcie model proves inadequate. Khaled and Vafai (2003) provided a comprehensive overview of applications involving heat and fluid dynamics in porous (biological) media. Ogulu and Amos (2007) delved into the effects of temporally-varying wall mass flux in hydro magnetic pulsatile Newtonian blood flow within a Darcie porous model of the cardiovascular system. They utilized a regular perturbation technique to investigate this phenomenon. This model has been used in various studies related to heat transfer in porous media, as elucidated by Pop and Ingham (2001). Notable contributions in this domain include the work of Preziosi and Farina (2002), who explored mass exchange using an extended Darcy model, and Vankan et al. (1997), who considered non-Darcy transport in blood-perfused tissue. Additionally, Sorek and Sideman (1986) analyzed blood flow in cardiac vessels using a Darcy-Forchheimer model. Recent research by Bhargava et al. (2007) employed the Darcy-Forchheimer model to investigate pulsating magneto-hydrodynamic blood flow and species diffusion within a porous medium channel. Of note, Joule dissipation plays a critical role as a volumetric heat source, garnering practical interest due to its relevance in various industrial applications. In a recent study, Sharma et al. (2013) delved into the intricacies of heat and mass transfer in a magnetic bio fluid flow through a non-Darcie porous medium, considering the Joule effect.

The aforementioned studies did not account for the influence of heat sources or sinks. However, recent pilot research suggests that the moisture content of the skin may alter the response of vascular endothelial cells to local heat, as demonstrated by KcLellan et al. (2009). In these investigations, it was observed that when the skin was kept dry during warming, the skin's blood flow response was notably smaller compared to when moist heat was utilized as the heating source (KcLellan et al., 2009). Existing literature provides some support for this notion. Petrofsky et al. (2009) proposed that the increased blood flow response observed in the skin might be attributed to the presence of moisture in the heat source, the faster rate of temperature increase in the skin, or a combination of both factors.In the realm of heat and mass transfer, Kandasamy et al. (2005) discussed these effects along a wedge with a heat source and concentration, while considering the first-order chemical reaction and the presence of suction/injection. Additionally, Sharma et al. (2007, 2008, 2011) explored various aspects of magneto hydrodynamic (MHD) free convective flow past an infinite vertical porous plate, including heat source/sink effects. Chambkha (2004) investigated unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Furthermore, Sharma et al. (2007, 2008, 2011) examined radiation effects in a free convective flow along a uniform moving porous vertical plate in the presence of a heat source/sink and transverse magnetic field.In a recent study, Sharma et al. (2014) explored the Soret and DuFour effects in an unsteady MHD mixed convective flow past an infinite vertical plate, considering Omics dissipation and heat source effects. Consequently, the primary aim of the present investigation is to examine biological fluid flow with heat and mass transfer considering its pulsatile hydro-magnetic rheological nature under the presence of viscous dissipation, Joule heating and a finite heat source through a Darcian porous medium.

2. Mathematical formulation

We examine the dynamics of an unsteady two-dimensional rheological bio-fluid flowing through a narrow channel in the presence of a porous medium. This fluid experiences viscous dissipation, Joule dissipation, and heat source effects. The channel consists of plates separated by a distance of 2H, where wall transpiration effects occur. The flow exhibits a pulsatile nature similar to blood circulation in the human cardiovascular system, driven by the heart's pumping action. The source parameter accounts for the influence of real pistons. Additionally, a transverse magnetic field (B0) is applied due to the bio-fluid's electrical properties.The porous medium is non-Darcian, and wall transpiration results in injection at (-H) and suction at (H). The energy equation incorporates Joule dissipation and heat source/sink terms. The bio-fluid is non-Newtonian, requiring the use of the Nakamura-Sawada model tailored for such fluids. Furthermore, the mass conservation equation considers the concentration C1 at the lower plate (-H) and C2 at the upper plate (H). In summary, these assumptions lead to the following system of governing equations.

Linear momentum equation

$$\frac{\partial u}{\partial t} + V_0\frac{\partial u}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial x} = V_B\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma\beta_0^2}{\mu}U - \frac{V_0}{k_p}U - bu^2 \qquad (2.1)$$

Energy equation.

$$\frac{\partial T}{\partial \tau} + V_0\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{V_B}{C_P}\left(\frac{\partial u}{\partial y}\right)^2 + S*(T-T_M) + \frac{\sigma B_0^2}{Pc_P}U^2 \qquad (2.2)$$

Concentration equation.

$$\frac{\partial C^I}{\partial \tau} + V_0\frac{\partial T}{\partial y} = D\frac{\partial^2 C^I}{\partial y^2} \qquad (2.3)$$

The corresponding boundary condition are

Y=-H; U=0; T=T_1 $C^I = C_I$;

Y=-H; U=0; $T = T_2$; $C^I = C_2$

Here's a revised version of the provided information: In the context of this study, the following parameters and variables are defined μB: Newtonian dynamic viscosity of the bio-fluid: Wall transpiration velocity, with V = Vo at the lower plate and V = -Vo at the upper plate.β: Upper limit of the apparent viscosity coefficient's: Longitudinal velocity component: Hydrodynamic pressure.kp: Hydraulic conductivity (permeability) of the porous material's: Density of the fluid: Horkheimer coefficient associated with the porous medium geometry: Dimensional time: Electrical conductivity of the bio-fluid.B0: Strength of the transverse magnetic field.α: Thermal diffusivity's: Specific heat capacity of the bio-fluid's: Source parameter: Temperature of the bio-fluid: Concentration of a species': Mass diffusivity of the species'/∂x: Longitudinal pressure gradient.

Introducing the following non-dimensional parameters.

$$U = \frac{u}{V_0},\ X = \frac{x}{H}, Y = \frac{y}{H}, T = \frac{V_0}{H}, P* = \frac{P}{\rho V_0}, \phi = \frac{T-T_m}{T-T_m}, \mathrm{Re} = \frac{HV_0}{V_\beta}, \mu M = \frac{\sigma B_0 H}{\rho V_0},$$

$$\lambda = \frac{K_P V_0}{V_B H}, NF = Hb, pr\frac{V_B}{\alpha}, Ec = \frac{V_0^2}{C_P(T_2 - T_m)}, Sc = \frac{HV_0}{D}, C = \frac{C^I - C_m}{C_2 - C_m}, S = \frac{S*}{V_0}$$

Equation (2.1) to (2.3) are reduced to the following non-dimensional

Momentum equation

$$\frac{\partial u}{\partial t}+\frac{\partial u}{\partial y}+\frac{\partial p}{\partial x}=\frac{1}{\mathrm{Re}}\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}-NmU-\frac{1}{\lambda}U-NfU^2 \qquad (2.4)$$

Energy equation

$$\frac{\partial \theta}{\partial t}+\frac{\partial \theta}{\partial y}=\frac{1}{prR}\frac{\partial^2 \theta}{\partial y^2}+\frac{Ec}{\mathrm{Re}}\left(\frac{\partial u}{\partial y}\right)^2+S\theta+EcNmU^2 \qquad (2.5)$$

Concentration equation

$$\frac{\partial c}{\partial t}+\frac{\partial c}{\partial Y}=\frac{1}{Sc}\frac{\partial^2 c}{\partial Y^2} \qquad (2.6)$$

The transformed boundary conditions become:

Y=-1: U=0; 0=-1 C=-1

Y=1 U=0; 0=1; C=1 (2.7)

In this context, we define the following dimensionless parameters and variables:

X: Dimensionless coordinate parallel to the bio-fluid flow. Y: Dimensionless coordinate transverse to the bio-fluid flow. U: Dimensionless transformed velocity component in the X-direction. P: Dimensionless transformed hydrodynamic pressure (omitted for analytical convenience).t: Dimensionless time. θ: Dimensionless temperature. Re: Dimensionless transpiration Reynolds number. Nm: Hydromagnetic parameter. λ: Darci an parameter representing permeability. Nf: Forchheimer parameter, characterizing quadratic porous drag. Tm: Characteristic temperature, defined as the average of T1 and T2.Cm: Characteristic concentration, calculated as the average of C1 and C2.Pr: Prandtl number. Ec: Eckert Number. Sc: Schmidt number. S: Dimensionless source parameter. Given that the fluid flow in this problem exhibits a pulsatile nature, we decompose the pressure gradient component into both a steady component and an oscillatory component, as follows:

$$-\frac{\partial P}{\partial x}=\left(\frac{\partial p}{\partial x}\right)_s+\left(\frac{\partial p}{\partial x}\right)_0 e^{iwt} \qquad (2.8)$$

First to solve the above equation (2.4) -(2.5) complied equation the pressure is redefined as.

$$-\frac{\partial p}{\partial x}=P_S=+P_0COS(w*t).$$

Were P_S = the static pressure component, P_0 = the oscillatory pressure component.

Method of solution

The nonlinear dimensionless partial differential equations described above, along with their associated boundary conditions, were numerically solved using the Explicit Finite Difference Technique. The finite difference equations corresponding to Eqs (2.4) to (2.6) are provided below:

$$U_i^{j+1} - U_i^{j+1} + \frac{U_i + l^i - U_i - 1^i}{2\Delta Y} + \left(P_S + P_0 Coswt\right) = \frac{1}{\text{Re}}\left(1 + \frac{1}{\beta}\right)U_{i+1}^2 - \frac{2U_1^i + U_{i-1}^i}{\Delta Y^2} -$$

$$\left(Nm + \frac{1}{\lambda}m\right)U_1^i - NF\left(U_1^i\right)^2 \tag{3.1}$$

$$\frac{\theta_i^{j+1} - \theta_i^{j+1}}{2\Delta t} + \frac{\theta_{i+1}^j - \theta_{i-1}^j}{2\Delta Y} = \frac{1}{\Pr \text{Re}} + \frac{\theta_{i+1}^j - 2\theta_i^j + \theta_{i-1}^j}{\Delta Y^2} + S\theta_i^j + \frac{Ec}{\text{Re}}\left(\frac{U_{i+1} - U_{i+1}}{2\Delta Y}\right)^2$$

$$+ NmEc\left(U_i^2\right) \tag{3.2}$$

$$\frac{C_i^{j+1} - C_i^{j+1}}{2\Delta t} + \frac{C_{i+1}^j - C_{i+1}^j}{2\Delta Y} = \frac{1}{Sc}\frac{C_{i+1}^j - 2C^i + C_{i+1}^j}{\Delta Y^2} \tag{3.3}$$

To derive the difference equations, the flow region is discretized into a grid or mesh consisting of lines aligned parallel to the Y and t axes. These mesh lines intersect at points known as nodes, where solutions for the difference equations are calculated. At each internal nodal point within a specific level (n), the finite-difference equations form a tri-diagonal system of equations. Solving these equations is achieved through the utilization of the Thomas algorithm (Hoffman, 1992). To establish the convergence of the finite difference scheme, computations are performed with slight variations in ΔY and Δt. During this process, negligible changes are observed in the values of u, T, and C. Additionally, after each iteration cycle, convergence checks are conducted to ensure that the convergence criterion is met at all points. Consequently, due to considerations of computational cost and accuracy, the chosen mesh size is deemed optimal.

4. Results and discussion

In the preceding sections, we have examined the characteristics of non-Newtonian bio-fluid flow in a non-Darcy porous medium influenced by hydro magnetism, heat source effects, and Joule heating. To gain a deeper understanding of this phenomenon, we conducted numerical simulations to analyses the distribution of velocity, temperature, and concentration across a range of dimensionless parameters. It's worth noting that the bio-fluid under consideration is blood, and we have adopted parameter values consistent with those found in the work of Sharma et al. (2013). To assess the precision of our results, we have carried out comparisons against established benchmarks. Comparisons have been conducted with the findings of Rawat et al. (2009), and the results are presented in Table 1. Remarkably, a strong concurrence is evident between both investigations, particularly in terms of the dimensionless velocity profile (U) concerning the transverse coordinate (Y).

Table 1. Comparison of present study and Rawat et al. (2009)

$\text{Re} = 0.5, \beta = 4, \omega = 8, \Pr = 10, P_0 = 7, \lambda = 5, NF = 0.002, Nm = 0.3, Ec = 6*109, \Pr = 21.$		
U (at t=0.5)		
Y	Rawat *et al* (2022)	Present study.
-1	0	0

-0.658554	0.477407	0.477453
-0.31707	0.792337	0.792369
0.31707	0.866090	0.866126
0.658537	0.572876	0.572933
1	0	0

Figure 1 illustrates the velocity profile at t=0.5 for various Reynolds number (Re) values. Notably, as the Reynolds number involves the transpiration velocity, an increase in Re corresponds to higher velocity (U) values, typically causing the peaks to shift towards the right. It's worth mentioning that a Reynolds number of unities signifies a creeping flow regime, where the inertial core found in higher-velocity porous flows has not yet formed, a concept discussed by Dibbs and Edwards (1984). In Figure 2, we examine the impact of the non-Newtonian parameter on the velocity profile. Lower values of this parameter indicate higher viscosity and consequently lower velocity values. Conversely, as this parameter increases, velocity also rises, and the peak velocity approaches its maximum as the parameter approaches infinity Figure 3 demonstrates the influence of the Darcy parameter (λ) on velocity profiles. Higher values of λ suggest reduced fatty deposits and obstructions within the fluid flow channel, resulting in lower resistance and higher velocities. The effects of the hydromagnetic parameter (Nm) are depicted in Figure 4. Increasing values of Nm lead to a downward shift in the velocity parabola. This phenomenon can be attributed to the retarding forces, specifically Lorentz forces, generated by the magnetic field, given the bio-fluid's established electrical properties. Such magnetic fields hold significant practical utility in regulating blood flows.

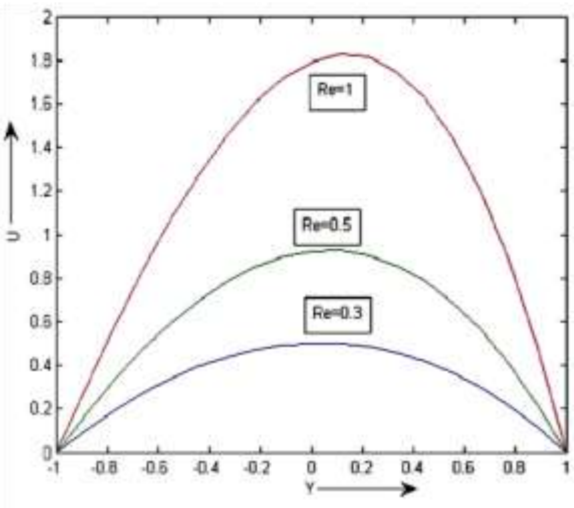

Fig.1. *U* versus *Y* for various transpiration Reynolds numbers (Re) at $t = 0.5$.

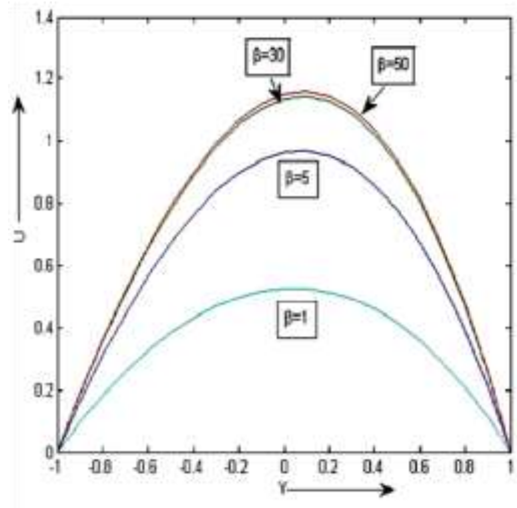

Fig.2. *U* versus *Y* for various non-Newtonian parameter values (β) at $t = 0.5$.

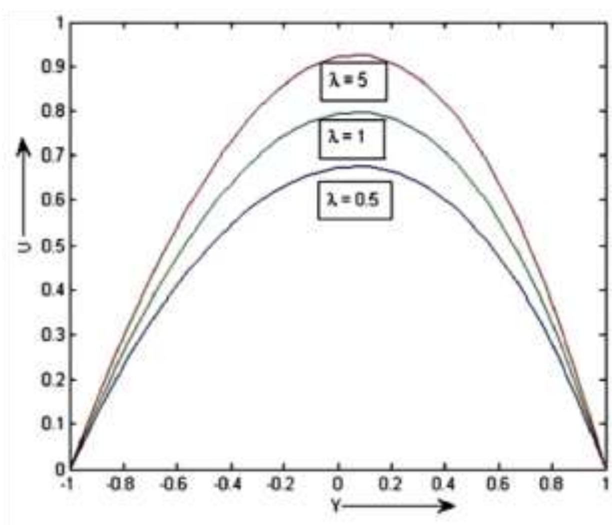

Fig.3. *U* versus *Y* for various Darci an parameter values (λ) at $t = 0.5$.

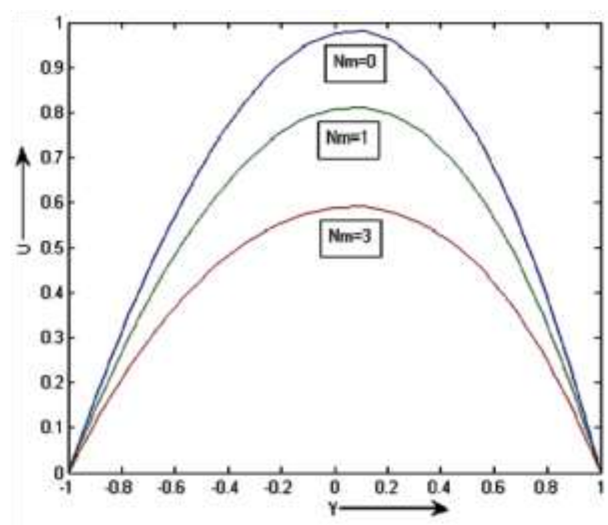

Fig.4. *U* versus *Y* for various hydromagnetic parameter values (*Nm*) at $t = 0.5$.

In Fig.5, The temperature is shown to be influenced by the hydro-magnetic parameter (Nm) at two Ec levels. It has been previously established that Nm inhibits temperature because of the Joule dissipation term in Eq. (2.4). At increasing Ec values, the Joule dissipation term plays the role of a volumetric heat source and gains importance. Figure 5a shows that temperature peaks diminish as Nm increases, whereas Figure 5b shows that bigger temperature values and more heat generation are produced by higher Ec values, which also cause an increase in oscillations because of the oscillatory character of the velocity profile. Fig. 6 displays the impact of the Prandtl number (Pr) on the temperature distribution for two distinct values of Ec. The ratio of heat diffusivity to momentum diffusivity is denoted by Pr. Momentum will diffuse more quickly in larger Pr fluids (Pr >1) than in heat. The numerical results indicate that when the Prandtl number increases, the temperature falls. This is because the thermal boundary layer thickness decreases when a fluid with a high Prandtl number has a relatively low thermal conductivity.

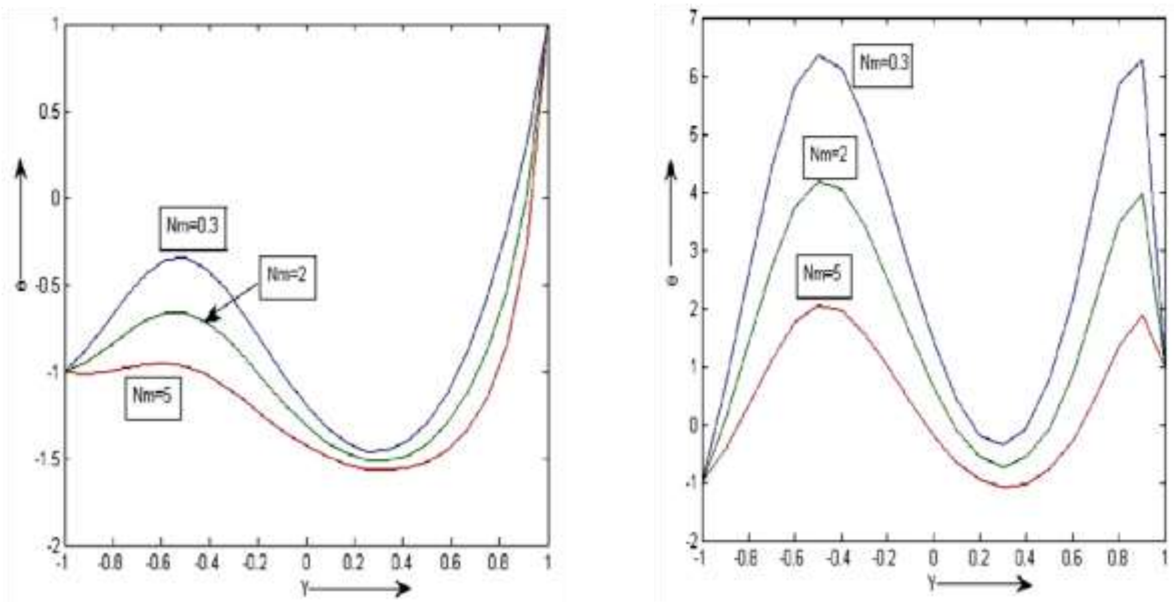

Fig.5. θ versus *Y* for various hydromagnetic parameter values (*Nm*) at *t = 0.5* for two Eckert numbers (a) Ec=*0.0006* (b) Ec=*0.006.*

Nomenclature

A – thermal diffusivity

B_0 – transverse magnetic field strength

b – Forchheimer coefficient related to the porous medium geometry

C – species concentration

c^1 – characteristic concentration

c_p – specific heat capacity of the bio-fluid

D – mass diffusivity of the species

Ec – Eckert number

k_p – hydraulic conductivity (permeability) of the porous material

Nf – Forchheimer (quadratic porous drag) parameter/ number

N_m – hydro-magnetic parameter

P – hydrodynamic pressure

*P** – transformed hydrodynamic pressure (* dropped for convenience in the analysis)

P_s – steady component of pressure gradient

P_0 – oscillatory pressure component

PR – Prandtl number

Re – transpiration Reynolds number

S – heat source parameter

Sc – Schmidt number

T – bio-fluid temperature

T T^{12} – characteristic temperature

t – dimensionless time

U – transformed velocity component in the X-direction

u – x - direction (longitudinal velocity)

V_o – wall transpiration velocity ($V = V_o$ at the lower plate and $V = -V_o$ at the upper plate)

X – dimensionless coordinate parallel to the bio-fluid flow

Y – dimensionless coordinate transverse to the bio-fluid flow

– upper limit of the apparent viscosity coefficient= rheological parameter (β)

– dimensionless temperature

– Darci an (permeability) parameter

$_B$ – Newtonian dynamic viscosity

– density of the fluid

– electrical conductivity of the bio-fluid

– dimensional time

– dimensionless angular frequency

P/X – longitudinal pressure gradient

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